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SCATTERING OF MASSIVE DIRAC PARTICLES BY A KINK-LIKE POTENTIAL

A. N. Ikot[1*], H. P. Obong[1], H. Hassanabadi[2],
T. M. Abbey[1], S. Zarrinkamar[3]

[1]Theoretical Physics Group, Department of Physics,
University of Port Harcourt, Choba, PMB 5323, Port Harcourt-Nigeria
[2]Physics Department, University of Shahrood, Shahrood, Iran
[3]Department of Basic Sciences, Garmsar Branch, Islamic Azad University, Garmsar, Iran

ABSTRACT

The scattering state of spin ½ particles with Kink-like potential is studied under the massive Dirac equation. We obtain the scattering states in terms of the hypergeometric functions and calculate the reflection coefficient (R) and transmission coefficient (T).

Keywords: Dirac equation, Kink-like potential, scattering state
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1. INTRODUCTION

The solutions of bound and scattering states of the relativistic and non-relativistic wave equation with constant mass and position dependent mass (PDM) have received a great attention in recent years [1-3]. The study of the quantum mechanical system with PDM has found many applications in different fields such as quantum dots [4], quantum liquids [5], the electronic properties of semiconductors [6], etc. [7 and references therein]. It is known that the full information of any quantum system is obtained from the determination of the eigenfunctions and the corresponding eigenvalues and in particular the scattering and bound-state solutions. There are now a wide variety of related studies. Candemir and Bayrak studied the scattering states of mass Dirac equation with asymmetric Hulthén potential [8]. The scattering states of Klein-Gordon equation with Kink-like potential have been investigated by Hassanabadi et al. [9]. Scattering and bound-state solutions of asymmetric Hulthén potential have been obtained by Arda et al. [10]. The bound-state and scattering states of Klein-Gordon equation with effective mass formalism have been studied by Arda and Sever [11]. Aydogdu et al. [12] reported the scattering and bound-states of massive Dirac equation with Wood-Saxon potential. Villalba and Rojas [13] proved that the relation between the bound-state energy eigenvalues and transmission resonances for the Klein-Gordon particle with Woods-Saxon potential resembles that of a Dirac particle. One of the potentials frequently used in quantum field theory and in the studies of topological classical theory is the Kink-like potential [14-15]. de Castro and Holt investigated the trapping of a neutral particle with Kink-like potential by mapping the problem into a Strum-Liouville equation [16]. Zarenia et al. studied the chiral states in bi-layer grapheme with Kink-anti-Kink-like potential [17]. Jia et al. also examined the solutions of the Klein-Gordon equation with the Kink-like potential [18]. The Kink-like potential is normally given in the form,

$$\begin{aligned} V(x) &= \frac{V_0}{2}\left[\theta(-x)\left(1-\tanh\left(-\alpha x\right)\right)+\theta(x)\left(1-\tanh\left(\alpha x\right)\right)\right] = \\ &= \frac{V_0}{2}\left[\theta(-x)\left(\frac{2e^{2\alpha x}}{1+e^{2\alpha x}}\right)+\theta(x)\left(\frac{2e^{-2\alpha x}}{1+e^{-2\alpha x}}\right)\right] \end{aligned} \tag{1}$$

where $\theta(x)$ is the step function and V_0 and α are positive parameters. The step potential limit can be obtained from the Kink-like potential by taking $\alpha \to 0,$ i.e.

$$\lim_{\alpha\to 0} V(x) \to V_0\left[\theta(-x)+\theta(x)\right] \tag{2}$$

In this work, we study the scattering states of the massive Dirac equation [19] under the Kink-like potential. Before starting the next section, we wish to address the very instructive papers [20-28] which investigate various aspects of the kink-like term and some related concepts including the topological defects, quantum fluctuations, etc.

2. DIRAC EQUATION WITH POSITION-DEPENDENT MASS

The Dirac equation in the relativistic units ($\hbar = c = 1$) is written as [19]

$$\left[i\gamma^{\mu}\partial_{\mu} - m(x)\right]\psi(x) = 0 \tag{3}$$

where $m(x)$ is mass of the Dirac particle that depends only on one spatial coordinate and the gamma matrices γ_x and γ_0 are taken as Pauli matrices $i\sigma_x$ and σ_z, respectively. Under the effect of external potential $V(x)$, the Dirac equation in one dimension becomes

$$\left[\sigma_x \frac{d}{dx} - \left(E - V(x)\right)\sigma_z + m(x)\right]\psi(x) = 0 \tag{4}$$

The four spinor $\psi(x)$ can be decomposed into the upper and lower components $\varphi(x)$ and $\chi(x)$. Thus, Eq. (4) becomes,

$$\frac{d\varphi(x)}{dx} = -\left[E - V(x) + m(x)\right]\chi(x) \tag{5}$$

$$\frac{d\chi(x)}{dx} = \left[E - V(x) - m(x)\right]\varphi(x) \tag{6}$$

By introducing the following combinations

$$\phi_1(x) = \varphi(x) + i\chi(x), \tag{8}$$

$$\phi_2 = \varphi(x) - i\chi(x) \tag{9}$$

and substituting Eqs. (8) and (9) into Eqs. (5) and (6), we get

$$\phi_1'(x) = -im(x)\phi_2(x) + i\left[E - V(x)\right]\phi_1(x) \tag{10}$$

$$\phi_2'(x) = -i\left[E - V(x)\right]\phi_2(x) + im(x)\phi_1(x) \tag{11}$$

The two components $\phi_1(x)$ and $\phi_2(x)$ respectively satisfy the second-order differential equations:

$$\frac{d^2\phi_1(x)}{dx^2} + \left[\left(E - V(x)\right)^2 - m^2(x) + i\frac{dV(x)}{dx}\right]\phi_1(x) = 0 \tag{12}$$

$$\frac{d^2\phi_2(x)}{dx^2} + \left[\left(E - V(x)\right)^2 - m^2(x) - i\frac{dV(x)}{dx}\right]\phi_2(x) = 0 \tag{13}$$

In this paper, we assume that the mass of the Dirac particle depends on spatial coordinate as [19] (see Fig. 2),

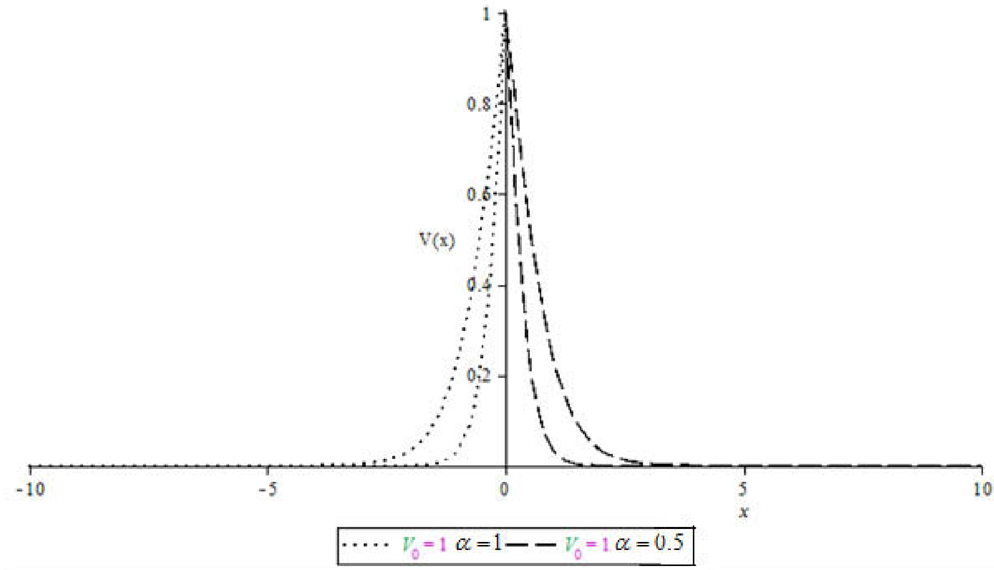

Figure 1: Plot of the Kink-like potential for $V_0 = 1$ and $\alpha = 1$ and 0.5 respectively.

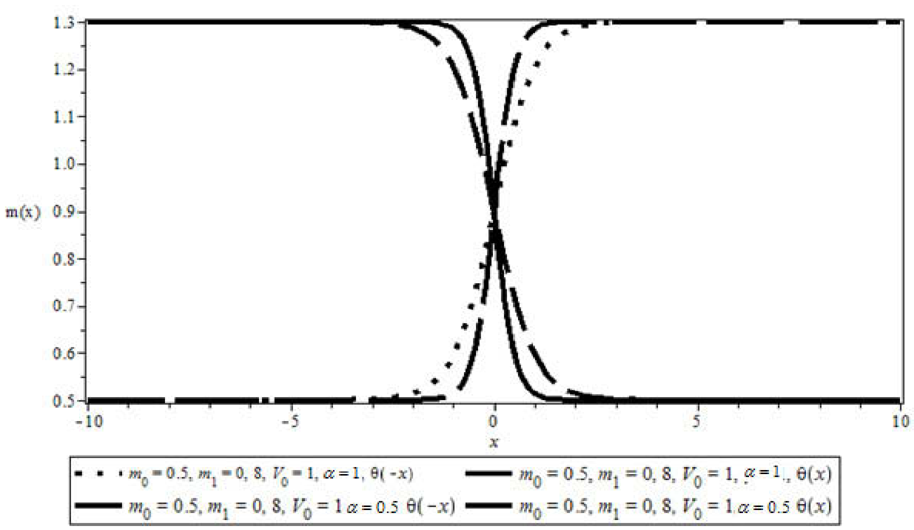

Figure 2: Mass function for various values of m_0, m_1, V_0 and α .

$$m(x) = m_0 + m_1 V(x) = m_0 + \frac{m_1 V_0}{2}\left[\theta(-x)\left(\frac{2e^{2\alpha x}}{1+e^{2\alpha x}}\right) + \theta(x)\left(\frac{2e^{-2\alpha x}}{1+e^{-2\alpha x}}\right)\right] \tag{14}$$

3. SOLUTION OF THE SCATTERING STATES OF THE DIRAC EQUATION IN THE PRESENCE OF THE KINK-LIKE POTENTIAL

3.1. THE SOLUTION IN THE NEGATIVE REGION ($x < 0$)

As we are searching for the scattering states of the equation for a Kink-like potential, we study the wave function. For $x < 0$, we substitute Eqs. (1) and (14) into Eq. (12) and have

$$\frac{d^2\phi_1(x)}{dx^2}+\left[\left(E-\frac{V_0e^{2\alpha x}}{1+e^{2\alpha x}}\right)^2-\left(m_0+\frac{m_1V_0e^{2\alpha x}}{1+e^{2\alpha x}}\right)^2+i\frac{d}{dx}\left\{\frac{V_0e^{2\alpha x}}{1+e^{2\alpha x}}\right\}\right]\phi_1(x)=0 \quad (15)$$

By applying the new variable $z_L = -e^{2\alpha x}$, Eq. (15) appears as

$$z_L\left(1-z_L\right)\frac{d^2\phi_1}{dz_L^2}+\left(1-z_L\right)\frac{d\phi_1}{dz_L}+\frac{1}{z_L\left(1-z_L\right)}\left[\omega_1 z_L^2+\omega_2 z_L+\omega_3\right]\phi_1(z_L)=0 \quad (16)$$

$$\omega_1=\left(\frac{E^2}{4\alpha^2}-\frac{2V_0E}{4\alpha^2}+\frac{V_0^2}{4\alpha^2}-\frac{m_0^2}{4\alpha^2}-\frac{2m_0m_1V_0}{4\alpha^2}-\frac{m_1^2V_0^2}{4\alpha^2}\right), \quad (17)$$

$$\omega_2=\left(-\frac{2E^2}{4\alpha^2}+\frac{2V_0E}{4\alpha^2}+\frac{2m_0^2}{4\alpha^2}+\frac{2m_0m_1V_0}{4\alpha^2}-\frac{2iV_0}{4\alpha}\right), \quad (18)$$

$$\omega_3=\frac{\left(E^2-m_0^2\right)}{4\alpha^2} \quad (19)$$

Taking the ansatz for the wave function as

$$\phi_1(z_L)=z^{\mu}\left(1-z\right)^{\upsilon}f(z) \quad (20)$$

and substituting it into Eq.(16), we obtain the hypergeometric-type equation

$$\begin{aligned}&z(1-z)f''(z)+\left[1+2\mu-\left(2\mu+2\upsilon+1\right)z\right]f'(z)-\\&-\left(\mu+\upsilon+\lambda\right)\left(\mu+\upsilon-\lambda\right)f(z)=0\end{aligned} \quad (21)$$

where,

$$\mu = \frac{i}{2\alpha}\sqrt{\left(E^2 - m_0^2\right)},$$

$$\upsilon = \frac{1}{2} - \sqrt{\left(\frac{1}{2} - i\frac{V_0}{2\alpha}\right)^2 - \frac{m_1^2 V_0^2}{4\alpha^2}} \tag{22}$$

$$\lambda = i\sqrt{\omega_1}$$

Comparing Eq. (21) with the general form of the second order differential equation of hypergeometric function

$$z(1-z)\frac{d^2\phi}{dz^2} + \left(c - \left(a+b+1\right)z\right)\frac{d\phi}{dz} - ab\phi(z) = 0 \tag{23}$$

we get the parameters *a*, *b*, *c* as follows

$$\begin{aligned} a &= \mu + \upsilon - \lambda, \\ b &= \mu + \upsilon + \lambda, \\ c &= 1 + 2\mu \end{aligned} \tag{24}$$

and the solution of Eq. (21) is the hypergeometric function given by

$$f(z) = {}_2F_1\left(a,b,c,z\right) \tag{25}$$

Using Eqs. (20) and (25), we obtain the wave function as

$$\begin{aligned} \phi_1(z_L) = Az^{\mu}\left(1-z\right)^{\upsilon}\, {}_2F_1\left(\mu+\upsilon-\lambda, \mu+\upsilon+\lambda; 1+2\mu; z\right) \\ B\, z^{-\mu}\left(1-z\right)^{\upsilon}\, {}_2F_1\left(-\mu+\upsilon-\lambda, -\mu+\upsilon+\lambda; 1-2\mu, z\right) \end{aligned} \tag{25}$$

We now consider the asymptotic form of the total wave function for $x < 0$. When $x \to -\infty$, $z \to 0$ and $(1-z) \to 1$, therefore Eq. (25) becomes

$$\phi_L(x \to -\infty) = A\left(-e^{2\alpha x}\right)^{i\sqrt{\omega_3}} + B\left(-e^{2\alpha x}\right)^{-i\sqrt{\omega_3}} \tag{26}$$

3.2. THE SOLUTION IN THE POSITIVE REGION ($x > 0$)

As in the previous section, substituting Eqs. (1) and (14) into Eq. (12) in view of the transformation $z_R = -e^{-2\alpha x}$, we obtain

$$z_R\left(1-z_R\right)\frac{d^2\phi_1}{dz_R^2}+\left(1-z_R\right)\frac{d\phi_1}{dz_R}+\frac{1}{z_R\left(1-z_R\right)}\left[\zeta_1 z_R^2+\zeta_2 z_R+\zeta_3\right]\phi_1(z_R)=0 \quad (27)$$

where,

$$\zeta_1=\left(\frac{E^2}{4\alpha^2}-\frac{2V_0E}{4\alpha^2}+\frac{V_0^2}{4\alpha^2}-\frac{m_0^2}{4\alpha^2}-\frac{2m_0m_1V_0}{4\alpha^2}-\frac{m_1^2V_0^2}{4\alpha^2}\right), \quad (28)$$

$$\zeta_2=\left(-\frac{2E^2}{4\alpha^2}+\frac{2V_0E}{4\alpha^2}+\frac{2m_0^2}{4\alpha^2}+\frac{2m_0m_1V_0}{4\alpha^2}+\frac{2iV_0}{4\alpha}\right), \quad (29)$$

$$\zeta_3=\frac{\left(E^2-m_0^2\right)}{4\alpha^2} \quad (30)$$

In order to obtain the scattering properties, we need to transform Eq. (27) into the form of hypergeometric equation. Therefore, we use the following transformation:

$$\phi_2=z^{\eta}\left(1-z\right)^{\delta}g(z) \quad (31)$$

and substituting it into Eq. (27) yields

$$\begin{aligned}&z(1-z)g''(z)+\left[1+2\eta-\left(2\eta+2\delta+1\right)z\right]g'(z)-\\&-\left(\eta+\delta+\beta\right)\left(\eta+\delta-\beta\right)g(z)=0\end{aligned} \quad (31)$$

where,

$$\eta=\frac{i\sqrt{\left(E^2-m_0^2\right)}}{2\alpha}, \quad (32)$$

$$\delta=\frac{1}{2}-\sqrt{\left(\frac{1}{2}+i\frac{V_0}{2\alpha}\right)^2-\frac{m_1^2V_0^2}{4\alpha^2}}, \quad (33)$$

$$\beta=i\sqrt{\zeta_1} \quad (34)$$

The solution of Eq. (31) is given in terms of the hypergeometric function as

$$\begin{aligned}\phi_1(z_R)=&Dz^{\eta}\left(1-z\right)^{\delta}{}_2F_1\left(\eta+\delta-\beta,\eta+\delta+\beta;1+2\eta;z\right)\\&Gz^{-\eta}\left(1-z\right)^{\delta}{}_2F_1\left(-\eta+\delta-\beta,-\eta+\delta+\beta;1-2\eta,z\right)\end{aligned} \quad (35)$$

Let us now consider the asymptotic form of the total wave function for $x > 0$. When $x \to +\infty$, $z \to 0$, $(1-z) \to 1$ and Eq. (31) becomes

$$\phi_R(x \to -\infty) = D\left(-e^{-2\alpha x}\right)^{i\sqrt{\zeta_3}} + G\left(-e^{-2\alpha x}\right)^{-i\sqrt{\zeta_3}} \tag{36}$$

The wave function in the negative and positive regions can be written as follows

$$\phi_1(x) = \begin{cases} A\left(-e^{2\alpha x}\right)^{i\sqrt{\omega_3}} + B\left(-e^{2\alpha x}\right)^{-i\sqrt{\omega_3}}, x \to -\infty \\ D\left(-e^{-2\alpha x}\right)^{i\sqrt{\zeta_3}} + G\left(-e^{-2\alpha x}\right)^{-i\sqrt{\zeta_3}}, x \to +\infty \end{cases} \tag{37}$$

The other component of the wave functions $\phi_2(z_L)$ and $\phi_2(z_R)$ can be obtained from Eq. (10) using Eq. (37) as follows:

$$\phi_2(x) = \begin{cases} A\left(\dfrac{E-(2\alpha)\sqrt{\omega_3}}{m(x)}\right)\left(-e^{2\alpha x}\right)^{i\sqrt{\omega_3}} + B\left(\dfrac{E+(2\alpha)\sqrt{\omega_3}}{m(x)}\right)\left(-e^{2\alpha x}\right)^{-i\sqrt{\omega_3}}, x \to -\infty \\ D\left(\dfrac{E-(2\alpha)\sqrt{\zeta_3}}{m(x)}\right)\left(-e^{-2\alpha x}\right)^{i\sqrt{\zeta_3}} + G\left(\dfrac{E+(2\alpha)\sqrt{\zeta_3}}{m(x)}\right)\left(-e^{-2\alpha x}\right)^{-i\sqrt{\zeta_3}}, x \to +\infty \end{cases} \tag{38}$$

The current density in terms of the one-dimensional Dirac spinors is defined as,

$$j = \frac{1}{2}\left[\left|\phi_1(x)\right|^2 - \left|\phi_2(x)\right|^2\right] \tag{39}$$

The electric current density in Eq. (39) can be written as $j_L = j_{in} - j_{reft}$ in the limit $x \to -\infty$, where j_{in} is the incident and j_{reft} is the reflected currents respectively. Likewise, as $x \to \infty$ the current density is $j_R = j_{trans}$, where j_{trans} is the transmitted current density. Substituting Eqs. (37) and (38) with $D = 0$ into Eq. (39), we find the reflection and transmission coefficients as,

$$R = \frac{\left(E+(2\alpha)\sqrt{\omega_3}\right)}{\left(E-(2\alpha)\sqrt{\omega_3}\right)}\left|\frac{B}{A}\right|^2, \tag{40}$$

$$T = \left|\frac{G}{A}\right|^2 \tag{41}$$

3.3. THE REFECTION AND TRANSMISSION COEFFICIENTS

In order to give the explicit expressions for the coefficients, we use the continuity conditions of the wave function and its first derivatives at $x=0$ i.e. $\psi_R(x=0)=\psi_L(x=0)$ and $\psi'_L(x=0)=\psi'_R(x=0)$, where prime denotes the derivative with respect to x. The former and the latter respectively yield

$$Ae^{-\pi\sqrt{\omega_3}}2^{\upsilon}U_1+Be^{\pi\sqrt{\omega_3}}2^{\upsilon}U_2=Ge^{\pi\sqrt{\zeta_3}}2^{\delta}U_3 \tag{42}$$

$$\begin{aligned}&Ae^{-\pi\sqrt{\omega_3}}(\alpha i)\sqrt{\omega_3}2^{\upsilon+1}U_1+Ae^{-\pi\sqrt{\omega_3}}\upsilon 2^{\upsilon-1}U_1+Ae^{-\pi\sqrt{\omega_3}}2^{\upsilon}\mu_1U_4-\\&Be^{\pi\sqrt{\omega_3}}(\alpha i)\sqrt{\omega_3}2^{\upsilon+1}U_2+Be^{\pi\sqrt{\omega_3}}2^{\upsilon-1}\upsilon U_2+Be^{\pi\sqrt{\omega_3}}2^{\upsilon}\mu_2U_5\\&\quad=Ge^{\pi\sqrt{\zeta_3}}(\alpha i)\sqrt{\zeta_3}2^{\delta+1}U_3+Ge^{\pi\sqrt{\zeta_3}}\delta 2^{\delta-1}U_3+Ge^{\pi\sqrt{\zeta_3}}2^{\delta}\mu_3U_6\end{aligned} \tag{43}$$

with

$$\mu_1=\frac{(\mu+\upsilon-\lambda)(\mu+\upsilon+\lambda)}{1+2\mu}, \tag{44}$$

$$\mu_2=\frac{(-\mu+\upsilon-\lambda)(-\mu+\upsilon-\lambda)}{1-2\mu}, \tag{45}$$

$$\mu_3=\frac{(-\eta+\delta-\beta)(-\eta+\delta-\beta)}{1-2\eta} \tag{46}$$

$$U_1={}_2F_1(\mu+\upsilon-\lambda,\mu+\upsilon+\lambda;1+2\mu;-1) \tag{47}$$

$$U_2={}_2F_1(-\mu+\upsilon-\lambda,-\mu+\upsilon+\lambda;1-2\mu;-1) \tag{48}$$

$$U_3={}_2F_1(-\eta+\delta-\beta,-\eta+\delta+\beta;1-2\eta;-1) \tag{49}$$

$$U_4={}_2F_1(\mu+\upsilon-\lambda+1,\mu+\upsilon+\lambda+1;2+2\mu;-1), \tag{50}$$

$$U_5={}_2F_1(-\mu+\upsilon-\lambda+1,-\mu+\upsilon+\lambda+1;2-2\mu;-1) \tag{51}$$

$$U_6={}_2F_1(-\eta+\delta-\beta+1,-\eta+\delta+\beta+1;2-2\eta;-1) \tag{52}$$

where we have used the relation $\frac{d}{ds}{}_2F_1(a,b,c;s)=\frac{ab}{c}{}_2F_1(a+1,b+1,c+1;s)$ in obtaining Eq. (43). Thus, the explicit relation for the coefficients A,B and G becomes,

$$\frac{B}{A}=\frac{\left[\left(\gamma_1\gamma_{10}+\gamma_1\gamma_{11}+\gamma_3\gamma_4-\gamma_3\gamma_5\right)U_1U_3+\gamma_1\gamma_{12}U_1U_6-\gamma_3\gamma_6U_3U_4\right]}{\left[\left(\gamma_2\gamma_{10}+\gamma_2\gamma_{11}+\gamma_3\gamma_7-\gamma_3\gamma_8\right)U_2U_3+\gamma_2\gamma_6U_2U_6-\gamma_3\gamma_9U_3U_5\right]} \tag{53}$$

and

$$\frac{G}{A}=\frac{\left[\left(\gamma_2\gamma_4+\gamma_2\gamma_5+\gamma_1\gamma_7-\gamma_1\gamma_8\right)U_1U_2+\gamma_2\gamma_6U_2U_4-\gamma_1\gamma_9U_1U_5\right]}{\left[\left(\gamma_2\gamma_{10}+\gamma_2\gamma_{11}+\gamma_3\gamma_7-\gamma_3\gamma_8\right)U_2U_3+\gamma_2\gamma_6U_2U_6-\gamma_3\gamma_9U_3U_5\right]} \tag{54}$$

where

$$\gamma_1=e^{-\pi\sqrt{\omega_3}}2^{\nu},\gamma_2=e^{\pi\sqrt{\omega_3}}2^{\nu},\gamma_3=e^{\pi\sqrt{\zeta_3}}2^{\delta},\gamma_4=e^{-\pi\sqrt{\omega_3}}\left(\alpha i\right)\sqrt{\omega_3}\,2^{\nu+1},\gamma_5=e^{-\pi\sqrt{\omega_3}}\nu 2^{\nu-1},$$

$$\gamma_6=e^{-\pi\sqrt{\omega_3}}2^{\nu}\mu_1,\gamma_7=e^{\pi\sqrt{\omega_3}}2^{\nu+1}\left(\alpha i\right)\sqrt{\omega_3},\gamma_8=e^{\pi\sqrt{\omega_3}}\nu 2^{\nu-1},\gamma_9=e^{\pi\sqrt{\omega_3}}2^{\nu}\mu_2,$$

$$\gamma_{10}=e^{\pi\sqrt{\zeta_3}}\left(\alpha i\right)\sqrt{\zeta_3}\,2^{\delta+1},\gamma_{11}=e^{\pi\sqrt{\zeta_3}}\delta 2^{\delta-1},\gamma_{12}=e^{\pi\sqrt{\zeta_3}}2^{\delta}\mu_3 \tag{55}$$

Now substituting Eqs. (53-54) into Eq. (40) and (41), we obtain the reflection R and transmission coefficient T as follows

$$R=\frac{\left(E+(2\alpha)\sqrt{\omega_3}\right)}{\left(E-(2\alpha)\sqrt{\omega_3}\right)}\left|\frac{\left[\left(\gamma_1\gamma_{10}+\gamma_1\gamma_{11}+\gamma_3\gamma_4-\gamma_3\gamma_5\right)U_1U_3+\gamma_1\gamma_{12}U_1U_6-\gamma_3\gamma_6U_3U_4\right]}{\left[\left(\gamma_2\gamma_{10}+\gamma_2\gamma_{11}+\gamma_3\gamma_7-\gamma_3\gamma_8\right)U_2U_3+\gamma_2\gamma_6U_2U_6-\gamma_3\gamma_9U_3U_5\right]}\right|^2, \tag{56}$$

$$T=\left|\frac{\left[\left(\gamma_2\gamma_4+\gamma_2\gamma_5+\gamma_1\gamma_7-\gamma_1\gamma_8\right)U_1U_2+\gamma_2\gamma_6U_2U_4-\gamma_1\gamma_9U_1U_5\right]}{\left[\left(\gamma_2\gamma_{10}+\gamma_2\gamma_{11}+\gamma_3\gamma_7-\gamma_3\gamma_8\right)U_2U_3+\gamma_2\gamma_6U_2U_6-\gamma_3\gamma_9U_3U_5\right]}\right|^2 \tag{57}$$

4. CONCLUSIONS

In the present paper, we solved the scattering problem of the one-dimensional Dirac equation with the Kink-like potential and reported the solutions hypergeometric functions. We obtain the refection and transmission coefficient using the derivative of the hypergeometric function and the continuity conditions. Our approach here offers one of the few examples where the Dirac equation is solved exactly with position-dependent mass and in an external potential. Finally, in addition to the fundamental importance in physics, the solutions obtained here may play a vital role in the study of hadrons for both theoretical and experimental physicists.

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