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Church's thesis as an empirical hypothesis

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The studies of Church's thesis (further denoted as CT) are slowly gaining their intensity. Since different formulations of the thesis are being proposed, it is necessary to state how it should be understood. The following is the formulation given by Church:

> [CT] The concept of the *effectively calculable function*[1] is identical with the concept of the *recursive function.*

The conviction that the above formulation is correct, finds its confirmation in Church's own words:[2]

> We now define the notion, […] of an *effectively calculable* function of positive integers by identifying it with the notion of a recursive function of positive integers […].

Frege thought that the material equivalence of concepts is the approximation of their identity. For any concepts F, G:

$$\text{Concept F is identical with concept G iff } \forall x\,(Fx \equiv Gx)\,[3].$$

The symbol Fx should be read as "object x falls under the concept F". The contemporary understanding of the right side of the above equation was dominated by the first order logic and set theory.

[1]Unless stated otherwise, the term *function* in this work always means *function defined on the set natural numbers*, that is, on natural numbers including zero. I would rather use the term *concept* than the term *notion*. Instead of *if and only if* will be used *iff*.

[2]This fragment comes form the seventh paragraph of the work 'Unsolvable Problem of Elementary Number Theory'. It is similar in the first paragraph although the formulation of this definition occurs in the third paragraph:' a positive integer function will be named effectively calculable if it is λ-definable in the sense of the second paragraph below'.

[3]Frege was not supposed to distinguish the signs ≡ and = (identity and equivalence). In his work of 1879 he used the sign ≡, while in the writings of 1893 and 1903 he used =. See: Jan H. Alnes, 'Sense and Basic Law V in Frege's Logicism', *Nordic Journal of Philosophical Logic*, 4(1999), pg. 1-30, in particular s. 4.

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This leads to regard (after Frege) the proposition expressing the material equivalence of concepts as identical with the proposition stating the equality of their extensions (sets).[4]

In the present work, the term hypothesis is understood as a proposition valued absolutely as true or false. However, the current state of investigations precludes precise determination of such values. Since the times of Popper, falsifiability is the criterion determining whether a theory or a hypothesis is scientific. Here, falsifiability will be understood broader as a possibility to prove the falsity of propositions. Although this is a necessary feature of empiricity, it is not sufficient for mathematical hypotheses fall under such category as well (e.g., continuum hypothesis). A hypothesis will be named empirical when it possesses empirical content. Empirical content of a hypothesis (theory) is a set of base propositions (observational) that can be deduced from the hypothesis by the use of logical methods. An observational proposition is a proposition whose logical value can be determined based on sensorial cognition (confrontation with reality). Such propositions contain observational terms. Their truth is established by contingent properties of the Universe, the world in which we live. From the epistemological point of view, one can say that these are a posteriori propositions.

1. A sample of the views on the empirical character of CT

Of the "fathers of calculability", Emil Post supported the understanding of CT as an empirical hypothesis. In his three page article "Finite Combinatory Processes. Formulation I." he gives his analysis of the process of calculation. Although independent, this method is almost identical with the Turing's analysis. Post's article reached the editorial office of Journal of Symbolic Logic on October 7, 1936 and, although it was published earlier, it is posterior to the famous work of the English logician. In the last chapter Post writes:

> The writer expects the present formulation to turn out to be logically equivalent to recursiveness in the sense of the Gödel-Church development.[…] Its purpose, however, is not only to present a system of a certain logical potency but also, in its restricted field, of psychological fidelity. In the latter sense wider and wider formulations are contemplated. On the other hand, our aim will be to show that all such are logically reducible to formulation I. We offer this conclusion at the present moment as a *working hypothesis*. And to our mind such is Church's identification of effective calculability with recursiveness.

[4] In Frege's system, the transition from the identity of the extension of notions to the identity of notions was possible based on the so called Basic Law V. This law was contradictory within Frege's system of the second order logic. Remarks on that matter are given in S. Shapiro, 'Foundations without Foundationalism', Oxford 2002, pg. 16-17.

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Here footnote number eight appears:

> […] Actually the work already done by Church and others carries this identification considerably beyond the working hypothesis stage. But to mask this identification under a definition hides the fact that a fundamental discovery in the limitations of the mathematicizing power of Homo Sapiens has been made and blinds us to the need of its continual verification.

And further on in the main text:

> Out of this hypothesis […] flows a Gödel-Church development. The success of the above program would, for us, change this hypothesis not so much to a definition or to an axiom but to a *natural law*.

The excerpts given above indicate that Post maintained the following:

- CT is neither a definition nor an axiom.[5]
- CT is a working hypothesis that demands continuing verification.
- CT was elevated by Church and others to the level of a valid law of nature.
- CT limits the (psychological) mathematical possibilities of man.

This understanding did not accord with that of Church. For him, the thesis was a definition. Therefore, in his review of the cited paper of Post, Church criticizes the understanding of CT as a working hypothesis that requires constant verification. He thinks that the notion of the effective calculability does not have precise sense and as a result CT does not have precise meaning as a working hypothesis. Calculability with the use of a machine (limited by the condition of finiteness) is supposed to be an adequate representation of an intuitive notion[6]. Then, according to Church, the need to assume such a hypothesis disappears. However, he did not relate to Post's view expressed in the quote cited. This is one of the few Church's comments on CT aside from the article in which it was formulated. It seems that later on he modified his standpoint in that matter. In 1940, he wrote the following[7]:

> Now a formal definition of effective calculability, for functions of positive integers, has been proposed by the author, […] and the adequacy of this definition to represent **the empirical notion** (A.O) of an effective calculation finds strong support in the recent result of Turing.

It is not clear what kind of definition CT is for him. In Reference 168 of his *Introduction to mathematical Logic* he distinguishes four types of definitions within logic: definitions that are abbreviations (a kind of synthetic definitions?),

[5]It is not clear how Post understood definitions. It seems that he understood them as what is sometimes called *synthetic definition*. Post's graduate student, Martin Davies, thought like his professor: '[…] how can we ever excluded the possibility of our being presented, some day (perhaps by some extraterrestrial visitors), with a (perhaps extremely complex) device or "oracle" that "computes" a noncomputable (in the sense of Turing; A.O.) function?', M. Davis, 'Computability and unsolvability', Dover, New York 1982, pg. 11.

[6]This review was published in: *Journal of Symbolic Logic*, 2 (1937) pg. 43.

[7]See: Alonzo Church, 'The Concept of Random Sequence', *Bulletin of the American Mathematical Society*, 46(1940), pg. 133.

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definitions that explicate the notation of the language (similar to real definitions)[8], semantic rules (metalanguage) that establish the interpretation of the system's language[9], definitions that extend the language of a formal system (they are a part of object language and must fulfil the conditions of correctness formulated by Leśniewski). It seems that CT can fall only under the second category of definitions due to the fact that Church attempted to provide justification of his thesis. If this is true, the opposition of Church towards Post's approach to CT seems strange because these two positions with regard to the character of CT could be reconciled.

Hao Wang was another logician with strongly philosophical inclinations who pointed out the empirical character of CT. In the book, *A survey of Mathematical Logic*, he writes[10]:

> It does seem that in the concept of effectiveness, there is a core in mechanical terms, and at the same time, there is an idealization which brings to infinity. Something which a physical object can do reliably and systematically would seem to be effective, no matter whether we understand the process or not. It would then appear to be an empirical question whether all effective functions are general recursive.

Thomas recalls a private conversation with Wang, in which the latter was supposed to say that the above quotation was aimed at making the empirical interpretation of CT probable and that he was not convinced of its correctness[11]. According to Wang, if the "empirical element" is removed (from the notion of effective calculability), the proof of CT is possible. If one takes CT with its empirical component, a real machine may be observed which computes a non recursive function. Thomas considers a hypothetical machine M that is supposed to compute a non recursive function. He points out that the machine M would have to have an infinite time of operation. This is due to the fact that each function of a finite domain of natural numbers is recursive and it demands infinite sampling time. Even if the "Zeno machine" is considered[12], a mathematician would need to follow an infinitely long proof in order to demonstrate that it computes a non recursive function. These arguments seem to be appropriate with respect to the machine M that exists in reality. Wang gives an example of such function suggested to him by Specker: f(n)=0 when on the

[8]In his context, Chuch does not speak about syntactic definitions. Regarding the second kind of definitions he mentions real definitions. However, he does not use this term and remarks that he wishes to avoid associations and presuppositions connected with this term.

[9]Church thought that one has to do with a formalized language as long as its interpretation is given. This idea is philosophically attractive but presently rarely respected.

[10]Hao Wang, op. cit., pg. 87.

[11]See: William J. Thomas, 'Church's Thesis and Philosophy', unpublished doctoral thesis, Case Western Reserve University, 1972, pg. 22.

[12]In regards to the paradoxes of Zeno of Elea. These machines perform the n+1 th step of an operation twice as fast as the n th step.

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n-th day (counting from today, February 1, 2005, for instance) an earthquake occured in the world, f(n)=1 if it did not occur. It is not well known how the recursiveness or the non recursiveness of the function f could be demonstrated. Also, it is problematic that *sensu stricto* one does not deal with a function defined in the entire set of natural numbers and the existence of such function would have to be postulated through the assumption of the existence of platonic objects. Functions that are calculable by means of physical systems will be called p-computable[13].

The version of CT that accords with Wang's view bears the name of the physical version of CT[14]:

[PCT] Any function that is p-computable is also computable by a Turing machine (T-computable)**[15]**.

A physical system mentioned above (real or potential) is defined by the following conditions: (1) its states occupy finite space, (2) its dynamics ends is real time, (3) dynamic is consistent with the laws of physics.

Computability by means of such a system means the following[16]:

> A computing machine is any physical system whose dynamical evolution takes it from one of a set of 'input' states to one of a set of 'output' states. […] The states are labeled in some canonical way, the machine is prepared in a state with given input label and then, following some motion, the output state is measured.

In the approximation we consider a physical system that can be assumed to compute a function in the sense given by Deutsch. Also, it is assumed that the behavior of the system is causal. Rosen stipulates[17] that in order to do theoretical science it is necessary to assume that one needs to simulate causal relationships occurring in physical (material) systems by the use of implications taking place between propositions which describe them. If the behavior of such a system is repeatable and it can be measured (observed), it is possible to seek a formal

[13]I have presented similar considerations in the following article: A. Olszewski, 'Teza Churcha a platonizm', *Zagadnienia Filozoficzne w Nauce*, 34(1999), pg. 96-100.

[14]Kreisl's version of the thesis states: *The behavior of any discrete physical system evolving according to local mechanical laws is recursive*. G. Kreisel, 'Mathematical Logic', [in:] 'Lectures on Modern Mathematics', Saaty (ed.), Wiley 1965, pg. 95-195. See Section 2.714.

[15]As one can see, the concept of a recursive function disappears form CT in favor of Turing machines. This corroborates with the fact that Turing machines are associated with the class of partially recursive functions. For interesting information on that matter see: 'A Speculative Church-Turing Theorem', U. Boker, N. Dersowitz. This paper provides an interesting formulation of CT: *All computational models are either equivalent or weaker with respect to Turing machines* (it refers to Church's formulation contained in the abstract published in 1935). As the authors postulate, the proof of CT is presented there.

[16]D. Deutsch, 'Quantum theory, Church-Turing principle and the universal quantum computer', *Proceedings of the Royal Society, Series A*, 400(1985), pg. 97-117.

[17]See R. Rosen, 'Effective Processes and Natural Law', [in:] 'The Universal Turing Machine. A Half-Century Survey', R. Herken (ed.), Springer, Wien, New York 1994, pg. 485- 498.

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system (computably recursive) that would model this system. If such a system were found and it were possible to cross from the physical system to a hypothetical formal one and vice versa, the system could be regarded as a realization of a formal system (and the formal system as a simulation (representation of physical system). This can be shown in the following graph[18]:

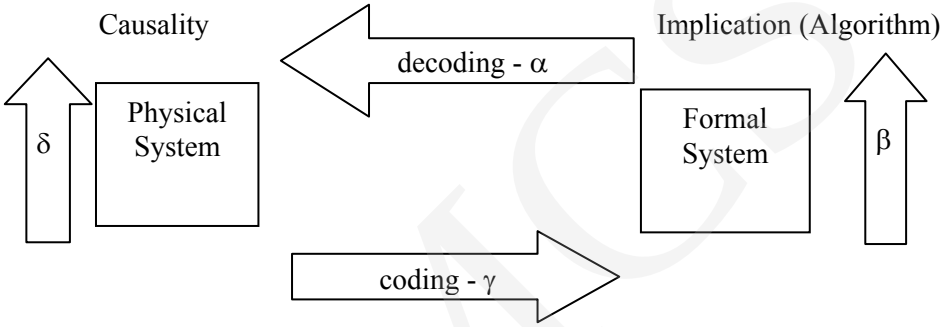

Fig. 1. Functional dependencies between the physical system and its formal description

If the arrows of the above graph are understood as functional dependences, a relation of modelling occurs between a given physical system and the formal system provided that the diagram commutes $\delta = \alpha \circ \beta \circ \gamma$[19]. According to Fitz, PCT is an extension of CT. A thesis inverse to PCT also bears empirical character because it is possible that one of the recursive functions (partially recursive) is not computable by a physical system. PCT has led physicists to consider physical systems exceeding the possibilities of Turing machines. This comprises Zeno machines (accelerating)[20], analog computation, quantum computation, quantum processes, machines utilizing relativistic effects, biological computation and others[21].

Robin Gandy must be also mentioned as a supporter of the empiricity of CT. He presents his concept in the work entitled "Church's thesis and Principles for Mechanisms"[22]. In his writings, he reflects on the computability performed by a machine as understood in the 19th century sense. As an example he mentions the "Babbage's Analytical Engine". The following are the limitations of the term "machine": a) **finitism** – analog machines are excluded from the consideration.

[18]Rosen, op. cit., pg. 491.

[19]See: Hartmut Fitz, 'Church's Thesis. A Philosophical Critique of the Foundations of Modern Computability Theory', Master's Thesis, Berlin, 2001, s. 94.

[20]The possibility of the existence of a physical system that performs an infinite number of steps (in its internal time) was supposed to be considered by Herman Weyl. P. H. Potgieter, 'Zeno Machines and Hypercomputations', submitted to *Elsevier Science*.

[21]For example see: P.H. Potgieter, *op. cit.*; J. Mycka, 'Empiryczne aspekty teorii obliczalności'; Fitz, *op. cit.*; R. Penrose, 'The Emperor's New Mind', Oxford Univ. Press 1989.

[22]Robin Gandy, 'Church's Thesis and Principles for Mechanisms', [in:] J. Barwise, H.J. Keisler, K. Kunen (eds.), "The Kleene Symposium', North-Holland, 1980, pg. 123-148.

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The only physical limitation is the lower limitation regarding atomic dimensions of the machine's parts and the upper limitation regarding the velocity of propagation (velocity of light). b) **discreteness** – the process of computation can be recorded with discrete terms, c) **determinism** – the machine's behavior is uniformly determined by its initial state.

Gandy demands that his description of a machine fits for every device of this kind: mechanical, electrical and notional. He formulates four principles that will define the machines: (I) there is a set S of descriptions of the states of a machine. This set is a subset of hereditarily finite sets (HF) (built above an infinite set of labels) together with a well defined transition function; the three following principles impose limitations on S and on the transition function $F:S\rightarrow S$; (II) a set theory level of machine construction is limited, that is $\exists k\ (S \subset HF_k)$; (III) for every description, there exists a limitation of the number of elements composing the machine; (IV) principle of local causality – every state belongs to a limited (local) former state. Gandy demonstrates that each function fulfilling these conditions (m-computable according to Kreisel) is computable by a Turing machine. Should any of these conditions be weakened, the machine will compute any function – it will admit of the free will.

[MCT] Any function that is computable by a finite, discrete and deterministic machine (m-calculable), is computable by a Turing machine (T-computable).

MCT can be rigorously proven and it is a mathematical theorem. Its empirical character consists in the assumption that each machine must fulfill the conditions (I-IV). However, this is a next *Thesis*[23]. Gandy formulates his argument as follows:

[G1] **Gandy's Thesis**: Any discrete, deterministic and mechanical device fulfills conditions (I-IV).

[G2] **Gandy's Theorem**: Any function computable by a device fulfilling conditions (I-IV) is also T-computable.

[G3] **Conclusion**: MCT[24].

Generally, it can be stated that if this formulation of CT has a strict proof, there exists a prior thesis, that has been accepted intuitively without a strict proof. In the subject literature, these assumptions bear the name of the *Central Theses* in support of CT.

Kleene has formulated two additional heuristic arguments in favor of CT. The first of them (a lack-of-counterexample argument) says that CT is true due to the fact that each effectively computable function as well as each effective operation defining a function from other functions turned out to be general recursive.

[23]The phenomenon of the *theses* is popular within science. Shagrir points out that there exists a problem with the interpretation of Gandy's machine. See Shagrir, op. cit., 234-236.

[24]See Gandy, op. cit., pg. 126 and 123.

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Various functions and their respective classes were investigated. It was the aim of these investigations to cover all known types of functions. Research methods preclude the situation in which it would be possible to find an effectively calculable function which cannot be converted into a recursive function by the use of these methods. Kreisel[25] gave an example of such a function. A constructively correct formal system of arithmetic is given and its proofs are constructively enumerated. A function f(n) is defined where f(n)=0 when the conclusion of the nth proof either does not have an existential form or it does but the proof does not specify the witness for this quantification: f(n)=(m+1) when the conclusion of the proof has an existential form and the proof specifies m as a witness. It turned out that the function is mechanically computable. This example is interesting (as Oddifreddi notices after Kreisel) there occurs 'the passage between a formal derivation […] and the corresponding mental, namely the proof expressed by the derivation'. Odifreddi thinks that this definition does not even make sense from the point of view of the set theory[26]. A Polish logician, Józef Pepis, who studied CT corresponded with Church on that matter, writes on the lack-of-counterexample argument[27]:

> In regards to this hypothesis, the above mentioned authors (Church and Turing; emphasis by A.O.) do not provide any convincing arguments in its support but they rely only on an ***empirical*** **fact** (emphasis by A.O.) that there are no known "calculable" functions except for those that are recursive. […] Due to such state of affairs, the question of the complete solubility of the problem of decidability for the system of the narrower functional calculus remains **open**. (emphasis by A.O).

The second of Kleene's arguments (called the formulation convergence argument) invokes a certain kind of stability of the effectively computable function. Various formal expressions of this notion turned out to be equivalent. Kreisel stresses that, despite of the equivalence of formulations, neither the systematical error in these approaches nor the fact that the notion of interest (effective computability) does not appear among the equivalent notions cannot be excluded[28].

In a summary, it can be stated that the three versions of Church's Thesis: CT, PCT and MCT result in consequences containing observational terms. These are concepts, physical systems and machines, respectively. The second and partially the third have designates clearly belonging to the Universe. Concepts - mysterious objects functioning in the mind are the designates of the first term.

[25]Kreisel, *op. cit*., Section 2.35.

[26]P. Odifreddi, 'Kreisel's Church', [in:] 'Kreiseliana', P.Odiferddi (ed.). AK Peters 1996, pg. 389-415; in particular 399-400.

[27]Józef Pepis, 'O zagadnieniu rozstrzygalności w zakresie węższego rachunku funkcyjnego', Lwów 1937, pg. 169-170.

[28]See Kreisel, *op. cit.*, Section 2.715.

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Consequently, the observational propositions for h-computable functions have slightly different character. Observation will be the introspection and the analysis of notions. Its sensorial character consists in the necessary emergence of the (formal) description of the computation of a function. Kant has denoted it as sensorial forms of eyewitness, in particular, in the form of space. In general, if one takes any object (algorithm, physical system, finite machine) and one proves that a function computable by such an object is recursive, one obtains partial confirmation of an appropriate version of CT. However, to find a counterexample (in any of the three domains) will falsify an appropriate version of CT. This implies that no man (physical system, machine) should not be able to compute effectively the values of a non recursive function (for any argument).

2. Turing's and Church's Arguments in favor of CT

The justification of CT given by Church himself appears in the seventh paragraph of "An Unsolvable Problem of Elementary Number Theory"[29]. Also, this is the second argument in favor of CT.

[C1] Definition: A function f is effectively computable iff there exists a formal system F in which it is weakly representable; namely, one argument function f is weakly representable in a certain system F iff there exists formula A in the language of system F such that for each $n \in N$: $\vdash_F A(\underline{n})=\underline{m}$ iff $f(n) = m$.[30]

[C2] Central Church Thesis:[31] The sets of axioms and rules of inference are recursively enumerable and each rule of inference is recursive, it means that there is a recursive function g such, that $g(n,x) = y$, where y is (the Gödel number of) the formula that is obtained (in F) from formulas (premises) coded by number x, by the use of the nth rule of system F.

[C3] Conclusion (CT): Each function that is effectively computable is recursive.

This argument is of deductive character. CT emerges logically out of the premises accepted. However, Church does not provide any justification for [C2][32].

[29] *Journal of Mathematics*, 58(1936), pg. 345-363. Reprinted in Davis' antology, pg. 88-107.

[30] Oron Shagrir, *op.cit.*, *Minds and Machines*, 12(2002) demands that the function be representable. However, this condition is not satisfactory because each function is representable in a contradictory system and CT would be consequently false.

[31] The name comes from: W. Sieg, 'Step by Recursive Step: Church's Analysis of Effective Calculability', *The Bulletin of Symbolic Logic*, 3(1997), pg. 154-180.

[32] See W. Sieg, *op. cit*., pg. 165.

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Let us assume that there exists effectively computable and non-recursive function f. Let us attach all propositions of the form P(n,m) to the axioms of a rich enough arithmetic AR in the way that: $\vdash_{AR'} P(n,m)$ iff $f(n)=m$ where P is a new binary predicate. The axiomatic of the system will remain effective as well as the system. Since no new principle is enclosed, the set of rules is effectively enumerable and each rule remains effective. The function f would be weakly representable in the hypothetical arithmetic AR'. In such a case, CT is clearly false. As a result, one cannot exchange in [C2] the word "recursive" for "effective". In spite of the low probability, it is possible that there exists an effective and non recursive function. Consequently, Post, who interpreted CT in the categories of cognitive psychology thought that in order to verify the thesis it would be necessary to investigate all possible ways human mind might formulate a finite process[33]. An exhaustive analysis of time-space possibilities of symbolization and symbolic "manipulation" by man is necessary[34]. This analysis regards man as present in the Reality – the Universe. Kreisel spoke in a similar fashion as he suspected that in order to evaluate CT in regard to functions computable by a human being (h-computable), one has to construct a formal system that encompasses the entire mathematics[35]. Post postulated the formulation of the theory of conception of concepts. It seems that this science would be of empirical character and account for real properties of human mind. While writing on Gödel, Gandy speaks similarly[36]:

> Gödel's objection (that intelligence works sometimes non-mechanically though perhaps effectively; A. O) can only be properly justified by a theory of intelligence. As he admits, our present understanding of the human mind is far from being penetrating enough for the construction of such theory. For this purpose the knowledge provided by introspection, the history of ideas, experimental psychology, neurophysiology and artificial intelligence seems meager indeed. One can only keep an open mind.

The Gödel's objection concerned CT in a sense that he denied its psychological interpretation. He thought that the results obtained with CT need

[33]See Fitz, *op. cit.*, ss. 37-39. See E. Post, Appendix of 'Absolutely Unsolvable Problems and Relatively Undecidable Propositions', reprint [in:] Davis' anthology, pp. 340-433. In references 1 and 9, Post stresses the necessity to investigate all ways of symbolization by man.

[34]See Post, 'Appendix', pg. 426

[35]G. Kreisel, 'Which number theoretic problems can be solved in recursive progressions on Π^1_1 – paths through *O*?', *The Journal of Symbolic Logic*, 37(1972), pg. 311-334, particularly pg. 316. According to Kreisel, the existence of such system does not contradict Gödel's incompleteness theorem.

[36]Gandy, *op. cit.*, pg. 124.

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to be interpreted as limiting the possibilities of pure formalism in mathematics[37]. For him, the Turing's analysis was a sufficient support for CT. He insisted on the interpretation of CT in the following shape:

[GCT] Each function computable by a mechanical procedure is computable by a Turing machine (T-computable)[38].

However, Gödel maintained that there exists communication among people that involves the content (sense) of expressions and not only the combinatoric (time-space) relations between combinations of signs[39].

The goal of the above consideration was to demonstrate that Church arbitrarily assumed [C2] while in place of [C2], [C2'] should be used:

[C2'] Relation of provability of **any** system fulfilling [C1] is recursive.

Since the truth of such premise depends on real properties of any possible and appropriate systems, it has an empirical character.

Let us now investigate the Turing's argument taken from the article 'On Computable Numbers with an Application to the Entscheidungsproblem'[40]:

[T1] **Central Turing Thesis**: a human computer (computor) obeys three limiting conditions.

[T2] **Turing's Theorem**: Any function that is computable by a computor obeying the three limitations is computable by means of a Turing machine.

[T3] **Conclusion (CT)**: each function computable by a computor, is computable by a Turing machine[41].

The three conditions mentioned are: determinism (each configuration of the computational process determines strictly the next step of calculations), limitation (this regards the number of directly recognizable symbolic configurations and the number of machine's internal states), locality (only directly observable configurations can be altered while the distance of newly observed configurations form those observed directly is limited).

There is a certain empirical motive in Turing's analysis of computability by man (it turned out to be very convincing for Gödel) that he assumed to be [T1] in his argumentation. The limitation imposed on the computor have to correspond to real limitations imposed by nature on man who performs computations. Is it really so? Gödel has criticized Turing's analysis with respect to the finite number of the computor's internal states. Gödel pointed out that

[37]See Postscriptum in Davis' antology, pg. 71-73

[38]See Hao Wang, 'From Mathematics to Philosophy', Routledge & Keagan, New York, 1974, pg. 84.

[39]K. Gödel, 'Über eine bisher noch nicht benützte Erweiterung des finiten Standpunktes', *Dialectica*, 12(1958), pg. 280-287. In his conversation with Wang, Godel pointed out that two premises need to be added to Turings's argument for correctness: 1. No matter independent mind exists; 2. The brain functions as a digital computer. See Wang, *op. cit.*, pg. 326.

[40]*Proceedings of the London Mathematical Society*, (2)42 (1936), ss. 230-265. Reprint [in:] Davis, pp. 115-154.

[41]Shagrir, *op. cit*. pp. 224-226.

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although the number of these states is finite, their number approaches infinity because mind is of its nature dynamical and not static[42]. This objection of Gödel is clearly of empirical origin for it refers to factual properties of mind. Currently, it is being stressed that the mechanistic assumption is presupposed by Turing's analysis[43].

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[42] See Davis, *op. cit.*, pg. 73.
[43] See Fitz, *op. cit.,* pg. 22.