Queuing in terms of complex systems

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Abstract − Limited resources are a natural feature of most real systems, both artificial and natural ones. This causes the need for effective management of access to existing resources. In this area, queuing systems are of special application. However, they are treated as simple systems for which two states are characteristic: work underload and work on the border of thermodynamic equilibrium. This approach is reflected in existing queue management mechanisms, that need to keep them in one of two mentioned states. On the other hand, they should be considered from the point of complex systems view, for which the third operation states: overload state is natural as well. In order to be closer to this issue, in this paper the authors consider queues performance from the perspective of complex systems.

1 Introduction

All real systems including natural systems such as humans, animals, plants and artificial systems for example computer networks, sensor systems, artificial intelligence, etc. are characterized by limited resources. This property is visible when there are more tasks to be serviced by the system than it would result from its natural capabilities. From this perspective three states of operating such systems should be taken into account: underload state, the state of work on the border of thermodynamic equilibrium [1] and an overload state. In the underload state, the problem of queuing is usually ignored or implemented in the form of FIFO models (First Input First Output) [2].This approach is applicable in most of the currently existing computer systems and

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networks. The FIFO queue allows for systematizing and preparing data for sending, but in no way does it adapt to change conditions of system operating. In the case of the second system state, various types of queuing models as well as mechanisms of queues prevention are taken into account. The first of them analyze flows, priorities etc., in order to classify data and then ensure that they are being permanently serviced. The congestion prevention mechanisms use different models that are based on rejection of packets [**3, 4**]. The above mentioned mechanisms are designed to ensure operation of the system in the first two states. The situation in which the system is in an overload condition results in either unavailability of the system (suspension system) or an attempt to return to the first state by rapid dropping packets, tasks, etc. Thus, the accepted and widely used models of queuing refer only to simple systems.

Taking into account only solutions related to simple systems while modelling a queuing system a priori imposes a limit in the form of analyzing of microscopic parameters only. Meanwhile, all real systems should be described by models in which there is close interaction between micro-and macroscopic parameters. It should be remembered that even a small change in one part of the system can significantly affect the performance of the whole system. This state of affairs is easily observable especially in computer systems and networks, where a small congestion can lead to a significant decrease system performance or even degradation of its resources.

Additionally, natural systems have essential features associated with self-organization and self-adaption, which allow them to work in the situations of overload. Currently available solutions for artificial systems in this field are based on the mechanisms, which, as mentioned above, only try to remain in the underload state or close to the border of acceptable use of resources.

Another aspect of the queue operation is a feedback that directly affects the performance of queues. Each condition is the result of not only the current state but significantly earlier states. This property is an integral feature of complex systems and refers to the processes associated just with feedback.

This paper presents a new approach to the problem of queuing in the systems and networks based on the theory of complex systems taking account of feedback as an important mechanism affecting the efficiency of the queuing process.

2   Queue as a simple system

Simple system is a special case associated with the unlimited resources of the system and homogeneous processes. This means that a simple system can be characterized by the following thermodynamic features: there are permitted only states in thermodynamic equilibrium in the system, all allowed states of the system are equally-probable, the number of states in the system is finite, the entropy has an extensive nature and the interactions between states in the system are short-term. Additionally, systems possess simple linearity characteristics as a function of load, stationary character of processes, deterministic flow - laminar, no overload and collapse of the system, the static capacity

planning, processes independence from the system resources, the normal distribution in space and time, etc. Therefore, a system with unlimited resources can be modelled as a proverbial black box (Fig. 1).
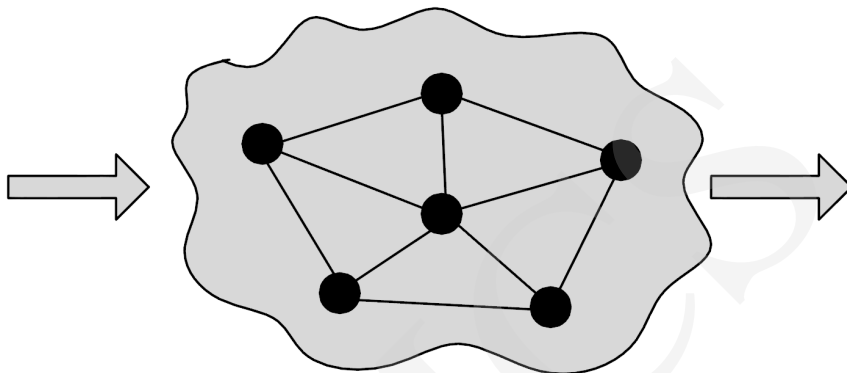

Fig. 1. Simple system as a black box.

From this perspective, the queuing system is represented by Little's law in terms of simple systems [**5**]. This law means that the level of the reserve in the process is the product of its performance and the length of the residence time. Thus, Little's law for the steady state is expressed by:

$$N = \lambda T, \tag{1}$$

where: λ is the intensity of the requests (tasks) inflow, T is the average time of the residence in the system (queue), and N represents the average number of tasks at a given time, that are located in the system (queue). This relationship concerning simple systems is presented in Fig. 2.

Thus, considering this system as a simple one we refer to the thermodynamic equilibrium, in which the processes are homogeneous and not undisturbed. Therefore this system can be treated as a black box, and its performance is described by the Malthus equation [**6**]:

$$X_M = \frac{dN}{dt} = rN, \tag{2}$$

where: r is the coefficient of growth (gain), and N is the number of tasks executed in the system.

In order to show the direct relationship between the expressions (1) and (2) let us consider the simple system remaining in the state of equilibrium. Let A denote the number of tasks that come into the system during observation, C is the number of tasks handled during the observation, while B is a time of engaging system. Taking parameters T, A, C and B into account, we can define the average values of the following system parameters: $\overline{\lambda}$, $\overline{X}$ and $\overline{S}$. Wherein $\overline{\lambda}$ is the intensity of the inflow
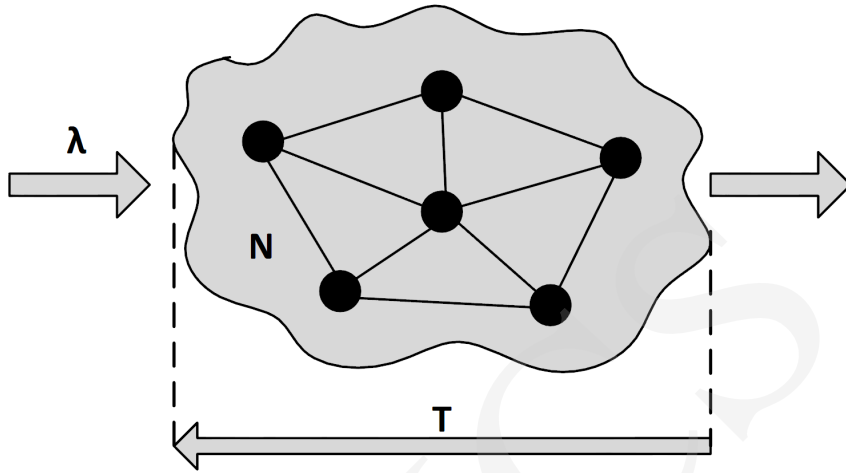
Fig. 2. Graphical representation of Little's law for the system in the steady state.

tasks to the system, $\overline{X}$ is a system performance, $\overline{S}$ is the time of handling task in the system and U is the coefficient of system resources utilization. They are defined respectively as:

$$\overline{\lambda} = \frac{A}{T},$$ (3)

$$\overline{X} = \frac{C}{T},$$ (4)

$$\overline{S} = \frac{B}{C},$$ (5)

$$U = \frac{B}{T} = \frac{B}{C}\frac{C}{T} = \overline{S}\,\overline{X} \leq 1.$$ (6)

Taking the parameters $\overline{\lambda}$ and $\overline{X}$ into account, it may be noted that in the general case, each system can be in one of three characteristic states at any point of time: 1 – underload $(\overline{\lambda} < \overline{X})$; 2 – matching between the number of tasks and system performance $(\overline{\lambda} = \overline{X})$; 3 – overload $(\overline{\lambda} > \overline{X})$.

The Malthus equation indicates that processes handling in the simple system is carried out by identical and mutually independent elements of a system in which there are no bottlenecks and therefore it can be described by Little's law as:

$$\overline{N}T = C\overline{R},$$ (7)

where: $\overline{N}$ is an average number of tasks in the system, and $\overline{R}$ is the average response time of the system. From equations (7) and (4) we obtain:

$$\overline{N} = \frac{C}{T}\overline{R} = \overline{X}\,\overline{R}.$$ (8)

Additionally, taking (1) into account, the parameter r can be defined as:

$$\frac{dX_M}{dN} = r = \text{const.} \tag{9}$$

Equation (9) defines the dynamics of the system and indicates that due to the unlimited resources, a simple system is insensitive to congestion and initial conditions.

3   Queues performance in terms of complex systems

The foundations of the complex systems theory were laid by Ludwig von Bertalanffy [7]. Originally it was a theory related to biology. However, it found promptly application in other areas of science and technology [8]. Features characteristic of complex systems are fractal geometry, small worlds theory, scale-free networks, power law, nonextensive statistics, non-linearity, thermodynamic disequilibrium, chaos theory, processes with a short and long term character [1, 9].

The fundamental feature of complex systems is self-organization. This term was introduced by Ashby in 1947 [10]. On this basis it can be concluded that the current state of the system is dependent on previous states and the input data. One of its features is that the global properties arise as a result of the interaction between elements of the system on a microscopic level (Fig. 3). Thus, the complex system is a dynamic system, in which changes taking place in its different areas (also locally) have often a significant impact on the operation on a macroscopic scale.
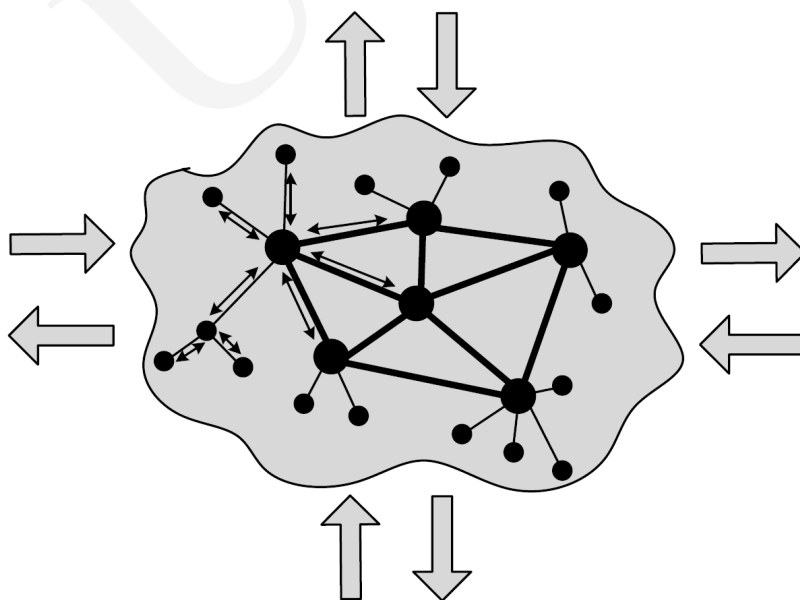

Fig. 3. Micro-and macroscopic relations in the complex system.

Space-time context of a physical model on the microscopic level can be shown by the queuing system models. In this case, each process handling in a complex system may be characterized by the microscopic parameters V_i, S_i, i and Z. Wherein V_i is the number of appeals (visits) to the i–th component of the system during handling task, where $0 \leq i < \infty$, and $0 \leq V_i < \infty$. On the other hand S_i is a time of handling task by the i–th component of the system, where in the general case $0 < S_i < \infty$, and Z is a thought time of a particular process.

Analyzing the systems performance (including queues), in which processes characteristic of complex systems occur, we should refer to the logistic equation of arbitrary order [11]:

$$X = rN \left(1 - \frac{N}{K}\right)^\phi,$$
(10)

where: K means the limited resources of the system, while ϕ is a self-organization process parameter of arbitrary order.

Considering the dependence of $r = 1/\overline{R}$, the above equation can be represented as:

$$X = \frac{N}{R}N \left(1 - \frac{N}{K}\right)^\phi.$$
(11)

Thus, it can be seen that the macroscopic r is the parameter determined by the microscopic parameters V_i, S_i, i and Z.

$$X = \frac{N}{\sum_i \overline{D}_i + Z} \left(1 - \frac{N}{K}\right)^\phi = \frac{N}{\sum_i S_i V_i + \overline{Z}} \left(1 - \frac{N}{K}\right)^\phi,$$
(12)

where: $\overline{D}_i$ is the time of handling task by any i-th component of the system.

There are three types of flows in the complex systems: laminar, reversible turbulent and irreversible turbulent (catastrophic or bad). The Malthus equation refers to a laminar flow while the logistic equation describes only a special case of reversal turbulent flow. Non-extensive thermodynamics indicates that the model of a complex system can be complete if the phenomena in the system can be of any nature, i.e. besides under-extensiveness there can also be super-extensiveness. Therefore, we propose a generalized version of the logistic equation, which relates to the whole family of system characteristics and can be written as:

$$X = \frac{N}{\sum_i S_i V_i + \overline{\overline{Z}}} \left(1 - \frac{N}{K}\right)^\phi = \frac{1}{\sum_i S_i V_i + \overline{\overline{Z}}} (1 - u)^\phi = X_M G.$$
(13)

Equation (13) shows that the performance of the executive complex system is a product of the simple system performance X_M and the deformation coefficient G, that modulates the system structure and processes of a non-degenerate system. Self-organization processes are characterized by the parameter ϕ. This is a product of the feedback factor, in other words sensitive to the initial conditions. It relates to the microscopic level of the system and can be any value in the range $-\infty < \phi < +\infty$. Equation (13) determines the susceptibility to deformation of the ideal space-time

structure, which leads to degradation of the simple system, and $G = (1 - u)^\phi$ is the deformation coefficient. Furthermore, you can notice that for $\phi = 0$ equation (13) corresponds to Malthus equation (2) and $\phi = 1$ leads to the original logistic equation. On the other hand, Malthus equation can be seen as a special kind of demarcation line that divides the plane $X(N)$ into two (above and below the line) diametrically different areas related to two different mechanisms of self-organization. The first characteristic, super-extensive area for $-\infty < \phi < 0$ and $0 < u < 1$, is located above the line and is self-managed only by the positive feedback. In turn, the sub-extensive area is located below the line of demarcation, $0 < \phi < +\infty$ and is connected with negative feedback.

4   Conclusions

Efficient management of available resources is the basis for operating of most systems, including computer technology. Currently available solutions based on queuing mechanisms try to handle appearing tasks in the deterministic way. When such a system reaches the thermodynamic equilibrium border, the rejection process takes place at random or in a deterministic way. This approach is characteristic of simple systems. On the other hand, natural systems which are complex ones can operate in an overload condition using self-organization and self-adaptation mechanisms. Therefore this paper is an introduction to further discussion about the possibility of creating effective mechanisms for the management of queuing systems in the terms of the complex systems theory.

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